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Determinant of Optimal Insurance in a Univariate Context: The Case of a "Non-Pecuniary Risk"

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ABSTRACT

Using a model of univariate decision under risk, we analyze the demand insurance when there is a single source of risk: a non pecuniary risk insurable. We examine how the insurable non pecuniary risk affects the demand for insurance of the individual. We show that the determinants of the demand for insurance are not only the shape of the insurance premium as offered by (Bernoulli, Mossin-Smith 1968), the correlation between the insurable financial risk and uninsurable financial risks as shown by Doherty and Schlesinger (1983a) and the variation of the marginal utility of wealth with respect to the health status (Rey, 2003), but also the way in which the occurrence of the insurable non pecuniary risk affects the marginal utility of wealth.

Keywords: *Non Pecuniary Risk, Insurance, Risk Aversion, Marginal Utility*

1. INTRODUCTION

The literature of the economic optimal insurance was strongly influenced by three propositions. The first well-known proposition is the Bernoulli principle. It states that risk-averse agents will choose full coverage when the premium is actuarially fair (no loading factor). The second proposition (Mossin, 1968; Smith, 1968) asserts that less than full coverage is optimal when the premium is loaded. The third proposition derives from Arrow (1963). It shows that a risk-averse agent will prefer a franchise contract to a coinsurance contract.

Doherty and Schlesinger show that these propositions do not necessarily hold in the presence of an uninsurable financial background risk. They show that sufficient conditions for the validity of propositions depend on the correlation between insurable and uninsurable background risk. Some papers analyzing the impact of background risk on insurance theory have been proposed later and represent significant progress (Doherty and Schlesinger, 1983b, 1985, 1990; Eeckhoudt and Kimball, 1992; Hau, 1999).

However, they all consider an uninsurable financial background risk and they use a one argument utility function. Rey (2003) takes into account this limit by using a bivariate utility function. She shows that the determinants of the demand for health insurance are not only the correlation between the health and uninsurable risks but also the variation of the marginal utility of wealth with respect to the health status. However, the model of Rey (2003) considers two types of risk that are insurable financial risk and other non-pecuniary uninsurable risk. Nevertheless, the individual may give largest weight a non-pecuniary risk that the financial risk weight. In fact, the losses of non- pecuniary risks are more important than financial risks. This observation is very common, in our days when environments are increasingly turbulent by the existence of natural disasters and political changes such as the revolutions of the Arab Spring of 2011 which produced a series of popular protest in some countries as in the case of: Tunisia, Egypt and Libya. These types of non-pecuniary risks have a strong correlation with economic, human risk (health individual) and social. The occurrence of these risks will be obvious consequences. That is why; the individual may give a greater weight to this type of risk (non-pecuniary) compared with other risks.

This paper extents the result obtained by Rey (2003) to this framework. We also consider a non-pecuniary risk (For example an accident risk) as an insurable risk, where the preferences of the individual are represented by a utility function uni-dimensional depend only wealth.

The objective of this paper is to examine how the insurable non-pecuniary risk affects the demand for insurance of the individual, in this context we seek to evaluate the standard results of the optimal insurance. We show that the optimal insurance depends crucially on the way in which the occurrence of the insurable non-pecuniary risk affects the marginal utility of wealth.

The remainder of the paper is organized as follows: the next section introduces the model. The section that follows examines the optimal coinsurance contracts. The last section concludes.

2. THE MODEL

We consider an individual who derives utility from wealth w. We use a Von Neumann Morgenstern one-argument utility function $u(w)$. We assume for u standard concavity assumptions:

$u_1 > 0$ and $u_2 > 0$ (the marginal utilities with respect to each argument are strictly positive).
$u_{11} < 0$ and $u_{22} < 0$ (the individual is risk averse towards a single risk on each argument of u)

The agent has an initial wealth w_0. The realization of a non-pecuniary risk translates into a loss of utility, passing from u to v such that $u > v$. We denote $v(W)$ the utility function, when the insurable loss occurs, with $v(W) < u(W)$. Indeed, the realization of the uninsurable loss always decreases the utility of wealth risk-averse preferences (Cook and Graham (1977)).

We assume for V standard assumptions: $v_1 > 0$, $v_2 > 0$, $v_{11} < 0$, $v_{22} < 0$.

3. OPTIMAL COINSURANCE CONTRACTS

To examine the optimal coinsurance contracts, we consider a coinsurance contract in which the insurance reimburses $I = \alpha(u - v)$ if non pecuniary risk occurs. The premium for insurance level α is $P(\alpha) = (1 + m)p\alpha(u - v)$: where m is the load factor, m ≥ 0.

$(u - v)$: denote non-pecuniary loss.

Two states of nature can appear. Utility levels and probabilities of occurrence are characterized as follows:

State 1: $u\ [w_0 - p(x)]$ no loss occurs $(1 - p)$

State 2:

$v\ [w_0 - p(x) + \alpha(u - v) - (u - v)]$

non-pecuniary risk occurs p

We note p the probability of occurrence of state i (i = 1...2)

Where p (resp. $(1 - p)$) denotes the probability of occurrence of the insurable non pecuniary risk (resp. no loss occurs).

The optimal level of insurance is solution of the following program:

$$\max_{\alpha}\ E(\alpha) = pv\left(w_0 - P(x) - (1-\alpha)(u-v)\right) + (1-p)\,u\left(w_0 - P(x)\right)$$

Where:

$$W_1 = (w_0 - P(x) - (u - v)(1 - \alpha)),\quad W_2 = (w_0 - P(x))$$

Equation (1) writes :

$$\max_{\alpha} E(\alpha) = pv\,(W_1) + (1 - p)\,u\,(W_2)$$

To examine the optimal coinsurance contracts, we consider in turn two situations: the situation where the premium is actuarially fair $(m = 0)$ and the situation where the premium is loaded $(m > 0)$.

3.1 The premium is actuarially fair

$: P(x) = p\alpha(u - v)$

In this part, we examine optimal coinsurance contracts where the premium is actuarially fair. The actuarially fair insurance premium is such premium when the expected loss of the insurance company equals exactly the revenue from insurance premium $(m = 0)$.

The analysis now focuses on the impact of insurable non-pecuniary risk on the request of the individual insurance: Individual Will it increase, decrease, or maintain the insurance application if (m=0)?

The answer to this question is provided by analysis of equation of the first order.
Equation of the first order writes:

$$v'(\alpha) = p(1 - p)(u - v)v'(W_1) - p(1 - p)(u - v)u'(W_2)$$

From the equation, we obtain the following results:

First, consider the case where the premium is actuarially fair (m = 0).

It is easy to verify that the Bernoulli principle stating that risk averse agents will choose full coverage when the insurance premium is actuarially fair does not hold in all cases. If the occurrence of the uninsurable loss decreases the individual's marginal utility of wealth then less than full insurance is desirable, and if the occurrence of the uninsurable loss increases the individual's marginal utility of wealth thus, over-insurance is optimal .So the Bernoulli principle only holds in one case summed up in the following proposition 1:

Proposition 1:

If the premium for insurance is actuarially (m = 0), the individual chooses full insurance (α^*= 1) if the following condition is true:

- The occurrence of the insurable loss non pecuniary no effect on the individual's marginal utility of wealth: $(u_1 = v_1)$.

Proof: See Annex 1.

We show that, contrary to the result of Bernoulli, the insured may choose a partial insurance even if the insurance premium is actuarially when the individual will ensure against non-pecuniary risk in the case the occurrence of the uninsurable loss decreases the individual's marginal utility

The proposition suggests that the demand for insurance depends on the impact of non-pecuniary risk affects the marginal utility of wealth.

The premium is loaded:

In this party we examine the individual's optimal insurance demand when the premium for insurance is loaded $(m > 0)$.

Let us now turn to the case where the premium is loaded (m > 0). The Mosin–Smith proposition that asserts that less than full coverage is optimal when the premium is loaded only holds under restrictive conditions.

Proposition 2:

If the premium for insurance is loaded $(m > 0)$, the individual chooses less than full insurance (α^*< 1) if one of the following conditions is true:

- The occurrence of the insurable loss non pecuniary no effect on the individual's marginal utility of wealth: $(u_1 = v_1)$.
- The occurrence of the uninsurable loss decreases the individual's marginal utility of wealth $(u_1 > v_1)$.

Proof: See Annex 2

Contrary to Mossin-Smith (1968), Rey (2003), the determinants of the demand for insurance depend not only on the shape of the insurance premium, and the correlation between the insurable and the insurable loss risk and the variation of the marginal utility of wealth with respect to the health status, but also on the way in which the occurrence of the insurable non-pecuniary risk affects the marginal utility of wealth.

Interpretation:

Proposition **1** and **2** shows the importance of the knowledge of the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth for determine the optimal insurance policy.

From the two propositions we can say that contrary to financial risk , the shape of the insurance premium (actuarial or loaded) does not represent the most important determinant of insurance coverage in a united context changed when s' is a non-pecuniary risk. This result is of great importance in terms of economic policy.

In fact, offering different insurance premiums, the insurance aims to reduce the problem of anti selection. Example, high risk will choose full insurance, and the low risks will choose less than full insurance. We note that when the risk is non-pecuniary the individuals (low risk or high risk) will choose full insurance.
The insurance company in this case (as opposed to financial risk) unable to know the low risks and high risks.

The percentage of low risk requiring full insurance and pay more insurance premium will increase. However, the low risks are described in non-pecuniary risk.

Thus, we should expect an increase in expected profits of insurance companies. We deduce that the non-pecuniary insurable risk has a positive effect on the insurance company, but it has a negative effect for the insured. We should expect the disappearance of insurance contracts with high deductible and low premium insurance in the case of non-pecuniary risk.

4. CONCLUSION

In this article, we consider that the individual will ensure against a single non-pecuniary risk instead of a financial risk. We use a utility function to a variable function of wealth. In this context we seek to validate the standard results of optimal insurance. Bernoulli, Mossin - Smith (1968), Doherty and Schlesinger (1983), and Rey (2003). We have shown that it is difficult to obtain classical results of the theory of insurance in the case of health. We concluded that the determinants of the demand for insurance is not only the shape of the insurance premium, the correlation between health and uninsurable risks and changes in the marginal utility of wealth over health, but also how the occurrence of non-pecuniary insurable risk affects the marginal utility of wealth . This result can be used to amend the standard results of optimal insurance.

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APPENDIX 1:

The premium is actuarially: $P(x) = \alpha(u-v)p$

Two states of nature can appear. Utility levels and probabilities of occurrence are characterized as follows:

State 1: $u\ [w_0 - p(x)]$ no loss occurs (1-p)

State 2: $v\ [w_0 - p(x) + \alpha(u-v) - (u-v)]$ non-pecuniary risk occurs (p)

Under these assumptions, the expected utility of the agent is written:

$$E(u) = pv\big(w_0 - P(x) - (u-v)(1-\alpha)\big) + (1-p)u\big(w_0 - P(x)\big) \quad (1)$$

Where $P(x) = (1+m)p\alpha(u-v)$ denote the premium for insurance

The optimal level of insurance is solution of the following program:

$$\alpha^* = \arg\max_\alpha v(\alpha) = pv\big(w_0 - P(x) - (1-\alpha)(u-v)\big) + (1-p)\,u\big(w_0 - P(x)\big)$$

(2)

Where:

$W_1 = (w_0 - P(x) - (u-v)(1-\alpha))$, $W_2 = (w_0 - P(x))$

If $m = 0$ so $P(x) = \alpha(u-v)p$: denote the premium is actuarially.

Consequently, the optimal level of insurance is solution of the following program:

$$\alpha^* = \arg\max_\alpha v(\alpha) = pv\big(w_0 - \alpha(u-v)p - (1-\alpha)(u-v)\big) + (1-p)\,u\big((w_0 - \alpha(u-v)p\big)$$

(2.1)

First order condition writes: $\frac{dv(\alpha)}{d\alpha} = 0$

$$v'(\alpha) = p\big(-(u-v)p + (u-v)\big)\ v'\big(w_0 - \alpha(u-v)p - (1-\alpha)(u-v)\big) + (1-p)\big(-(u-v)p\big)\,u'\big((w_0 - \alpha(u-v)p\big)$$

(2.2)

Equation (2.1.1) can be written as follows:

$$v'(\alpha) = p(1-p)(u-v)v'\big(w_0 + (1-p)\alpha(u-v) - (u-v)\big) - p(1-p)(u-v)u'(w_0 - \alpha(u-v)p)$$

(2.2.1)

Equation (2.1.2) can be written as follows:

$$v'(\alpha) = p(1-p)(u-v)v'(W_1) - p(1-p)(u-v)u'(W_2)$$

(2.2.2)

Where:

$W_1 = (w_0 - \alpha(u-v)p - (u-v)(1-\alpha))$, $W_2 = (w_0 - \alpha(u-v)p)$

Second order condition writes: $\frac{d^2v(\alpha)}{d^2\alpha} < 0$

$$v''(\alpha) = p\big(-(u-v)p + (u-v)\big)\big(-(u-v)p + (u-v)\big)\ v''\big(w_0 - \alpha(u-v)p - (1-\alpha)(u-v)\big) + (1-p)\big(-(u-v)p\big)\big(-(u-v)p\big)\,u''\big((w_0 - \alpha(u-v)p\big)$$

(2.3)

Equation (2.2) can be written as follows:

$$v''(\alpha) = p[(1-p)(u-v)]^2\big[v''\big(w_0 - \alpha(u-v)p - (1-\alpha)(u-v)\big) - u''\big((w_0 - \alpha(u-v)p\big)$$

(2.3.1)

The second order condition is satisfied because $v''(\alpha)$ is always negative.

Proof (1):

v (α) being concave, v '(α) = 0, so we have: $v'(W_1) = u'(W_2)$ **but** $W_1 \neq W_2$

Case (1):

If $v'(x) = u'(x)$

From the first order condition, we have $v'(W_1) = u'(W_2)$

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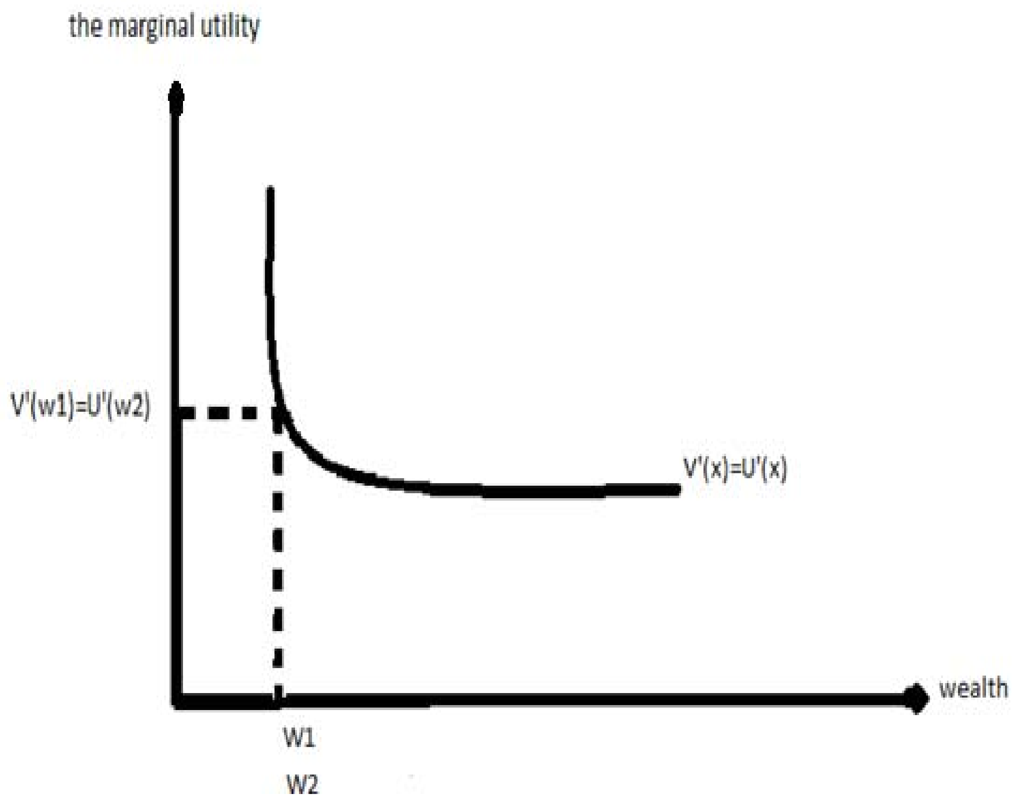
GRAPHIC 1:



It is easy to show from the graphic (1) that:

$W_1 = W_2$

$\left(w_0 + (1-p)\alpha(u-v) - (u-v)\right) =$
$(w_0 - \alpha(u-v)p)$

$\left(w_0 - p\alpha(u-v) + \alpha(u-v) - (u-v)\right) =$
$(w_0 - \alpha(u-v)p)$

$\alpha(u-v) = (u-v)$

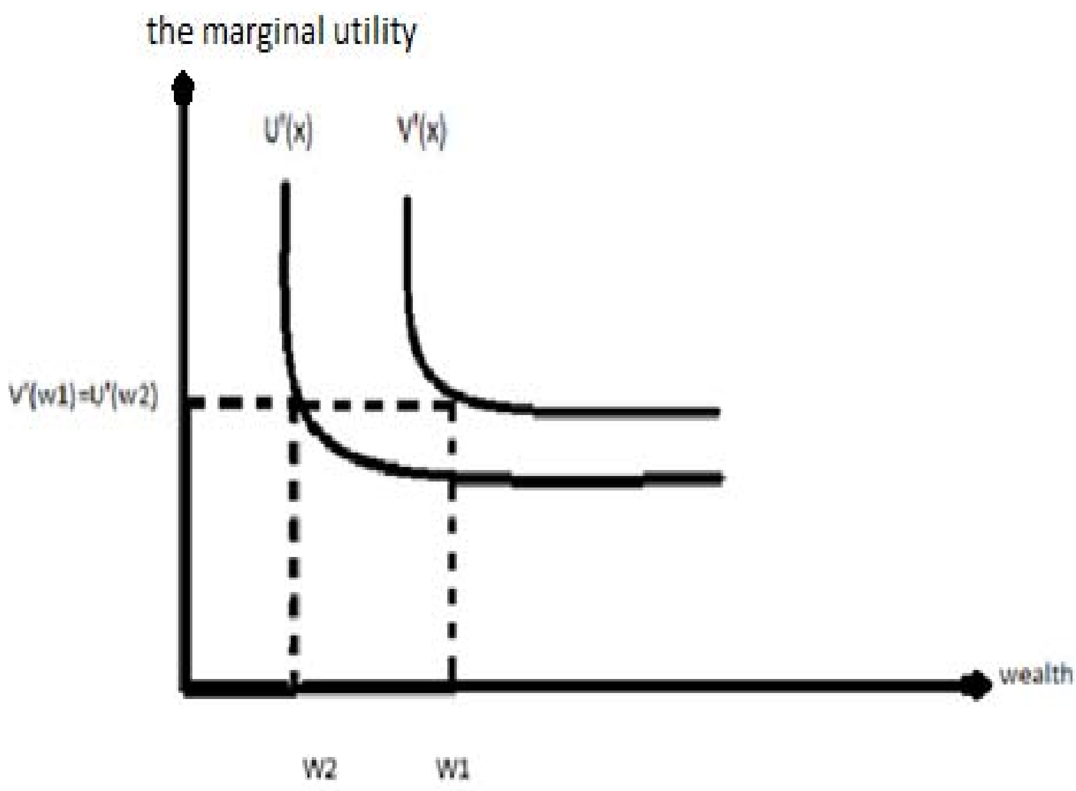
$\alpha^* = 1$: The full insurance is optimal.

Case (2) :

If $v'(x) > u'(x)$, et $v'' < 0, u'' < 0.$

From the first order condition, we have $v'(W_1) = u'(W_2)$

GRAPHIC: 2



It is easy to show from the graphic (2) that:

$W_1 > W_2$

$(w_0 + (1-p)\alpha(u-v) - (u-v)) > (w_0 - \alpha(u-v)p)$

$(w_0 - p\alpha(u-v) + \alpha(u-v) - (u-v)) >$
$(w_0 - \alpha(u-v)p)$

$\alpha(u-v) > (u-v)$

$\alpha^* > 1$: The over-insurance is optimal.

Case (3):

Si $v'(x) < u'(x)$,

From the first order condition, we have $v'(W_1) = u'(W_2)$

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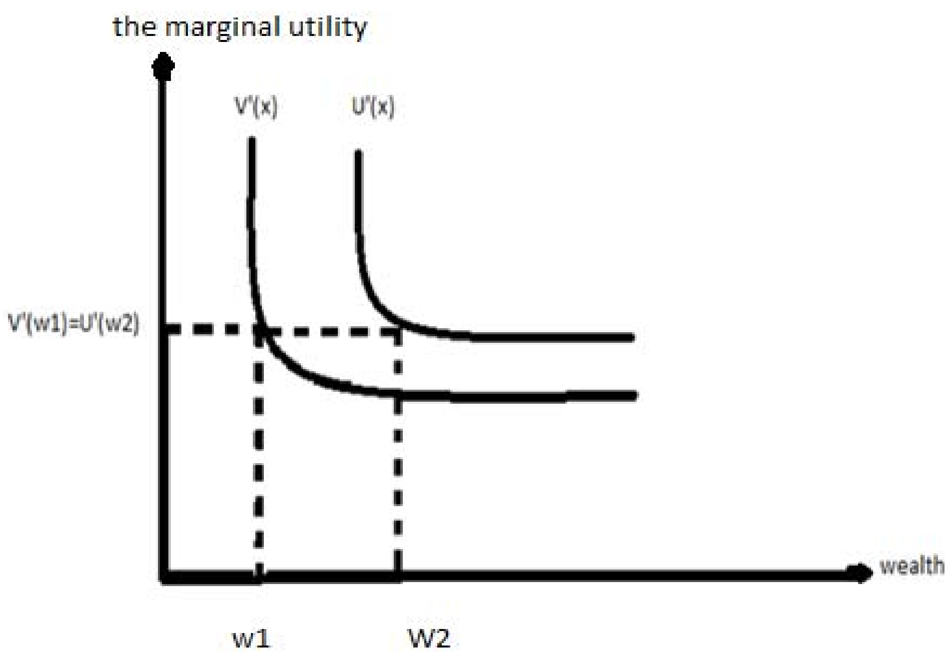
GRAPHIC: 3



It is easy to show from the graphic (3) that:

$W_1 < W_2$

$(w_0 + (1-p)\alpha(u-v) - (u-v)) < (w_0 - \alpha(u-v)p)$

$(w_0 - p\alpha(u-v) + \alpha(u-v) - (u-v)) < (w_0 - \alpha(u-v)p)$

$\alpha(u-v) < (u-v)$

$\alpha^* < 1$: The less than full insurance is optimal.

Appendix 1:

The premium is loaded: $P(x) = p\alpha(u-v)(1+m)$

Two states of nature can appear. Utility levels and probabilities of occurrence are characterized as follows:

State 1: $u\ [w_0 - (1+m)p\alpha(u-v)]$ no loss occurs $(1-p)$

State 2:

$v\ [w_0 - (1+m)p\alpha(u-v) + \alpha(u-v) - (u-v)]$ non-pecuniary risk occurs (p)

Under these assumptions, the expected utility of the agent is written:

$$E(u) = p\ v\,(w_0 - (1+m)p\ \alpha(u-v) - (1-\alpha)(u-v)) + (1-p)\ u\,((w_0 - (1+m)p\ \alpha(u-v)) \quad (3)$$

Where $P(x) = (1+m)p\alpha(u-v)$ denote the premium for insurance

The optimal level of insurance is solution of the following program:

$$max_{\alpha}\, v(\alpha) = p\ v\,(w_0 - (1+m)p\ \alpha(u-v) - (1-\alpha)(u-v)) + (1-p)\ u(w_0 - (1+m)p\alpha(u-v)). \quad (4)$$

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Where:

$W_1 = (w_0 - (1+m)p\,\alpha(u-v) - (1-\alpha)(u-v))$, $W_2 = (w_0 - (1+m)p\alpha(u-v))$

First order condition writes: $\frac{du(\tau)}{d\alpha} = 0$

$$v'(\alpha) = p(-(1+m)p(u-v) + (u-v))v'(w_0 - (1+m)p\alpha(u-v) - (1-\alpha)(u-v)) + (1-p)(-(1+m)p(u-v))u'(w_0 - (1+m)p\alpha(u-v)).$$

(4.1)

Equation (4.1) can be written as follows:

$$v'(\alpha) = (u-v)[1-p(1+m)]p\,v'[w_0 + \alpha((u-v)(1-p(1+m)) - (u-v)] - (1-p)p(1+m)(u-v)u'(w_0 - p(1+m)\alpha(u-v))$$

(4.1.1)

Second order condition writes: $\frac{d^2v(\alpha)}{d^2\alpha} < 0$

Prevue (2):

$v'(\alpha) = 0$, on a donc : $v'(W_1) > u'(W_2)$

$$v'(\alpha) = (u-v)[1-p(1+m)]p\,v'(W_1) - (1-p)p(1+m)(u-v)u'(W_2)$$

$$(u-v)[1-p(1+m)]p\,v'(W_1) = (1-p)p(1+m)(u-v)u'(W_2)$$

$$\frac{1-p(1+m)}{(1-p)(1+m)} = \frac{u'(w_0 - p(1+m)(u-v))}{v'(w_0 + (u-v)(1-p(1+m)) - (u-v))}$$

$$1-p(1+m) = 1-p-mp$$
$$(1-p)(1+m) = 1-p+m-mp$$

$$1-p-mp < (1-p)(1+m)$$

So :

$$\frac{1-p(1+m)}{(1-p)(1+m)} < 1$$

$$\frac{u'(W_2)}{v'(W_1)} < 1$$

:

$$v''(\alpha) = p(-(1+m)p(u-v) + (u-v))(-(1+m)p(u-v) + (u-v))v''(w_0 - (1+m)p\,\alpha(u-v) - (1-\alpha)(u-v)) + (1-p)(-(1+m)p(u-v))(-(1+m)p(u-v))u''((w_0 - (1+m)p\,\alpha(u-v))$$

(4.2)

Equation (4.2) can be written as follows:

$$v''(\alpha) = (u-v)^2[1-p(1+m)]p\,v''[w_0 + \alpha((u-v)(1-p(1+m)) - (u-v)] - (1-p)p(1+m)(u-v)^2u''(w_0 - p(1+m)\alpha(u-v))$$

(4.2.1)

The second order condition is satisfied because $v''(\alpha)$ is always negative.

Proof (1):

v (α) being concave, v '(α) = 0, so we have: $v'(W_1) = u'(W_2)$ **but** $W_1 \neq W_2$

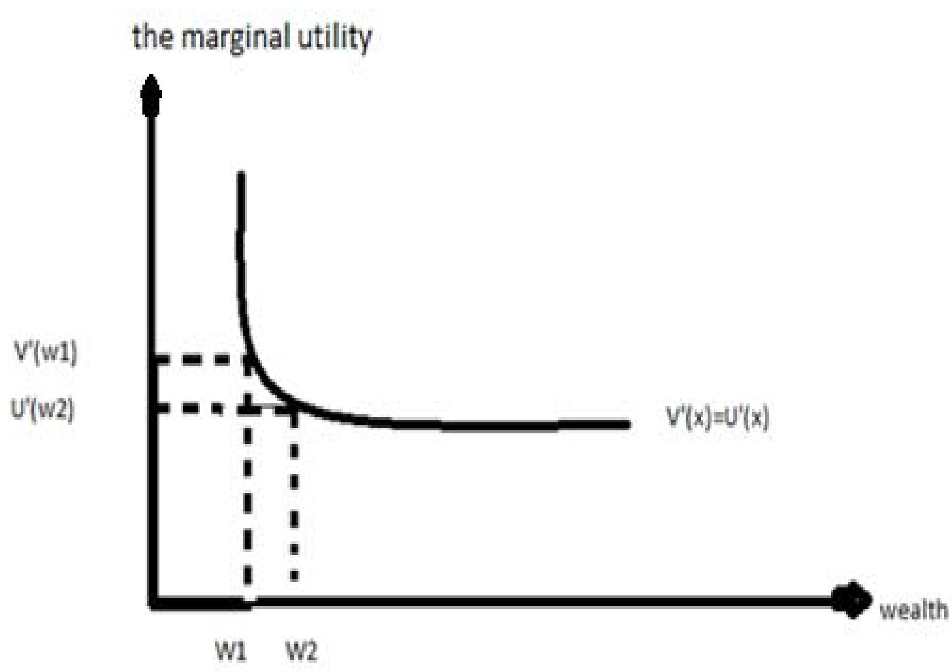
So with the first order condition (equation (4.1.1)) we can put assumption: $v'(W_1) > u'(W_2)$ Thus, three cases are possible:

Case (1):

If $v'(x) = u'(x)$

From the first order condition, we have $v'(W_1) > u'(W_2)$

GRAPHIC: 4



It is therefore easy to show that:

$W_2 > W_1$

$(w_0 - \alpha(u - v)p(1 + m)) > (w_0 + (1 - p(1 + m))\alpha(u - v) - (u - v))$

$(w_0 - \alpha(u - v)p(1 + m)) -> (w_0 - p(1 + m)\alpha(u - v) + \alpha(u - v) - (u - v))$

$\alpha(u - v) - (u - v) < 0$

$(u - v) > 0$

So $(\alpha - 1) < 0$, $\alpha < 1$

$\alpha^* < 1$: The less than full insurance is optimal.

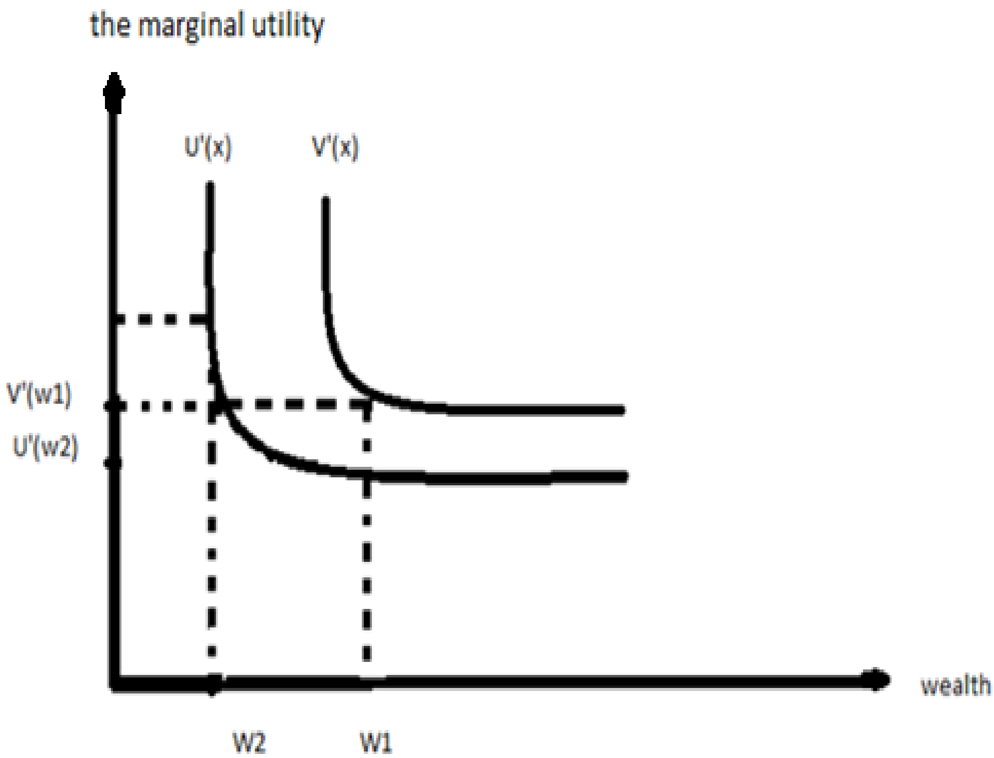
Case (2) :

If $v'(x) > u'(x)$, et $v'' < 0, u'' < 0.$

From the first order condition, we have

$v'(W_1) = u'(W_2)$

GRAPHIC: 5



It is therefore easy to show that:

$W_1 > W_2$

$(w_0 + (1-p)\alpha(u-v) - (u-v)) > (w_0 - \alpha(u-v)p)$

$(w_0 - p\alpha(u-v) + \alpha(u-v) - (u-v)) > (w_0 - \alpha(u-v)p)$

$\alpha(u-v) - (u-v) > 0$

$\alpha > 1$

$\alpha^* > 1$: The over-insurance is optimal.

Case (3):

If $v'(x) < u'(x)$,

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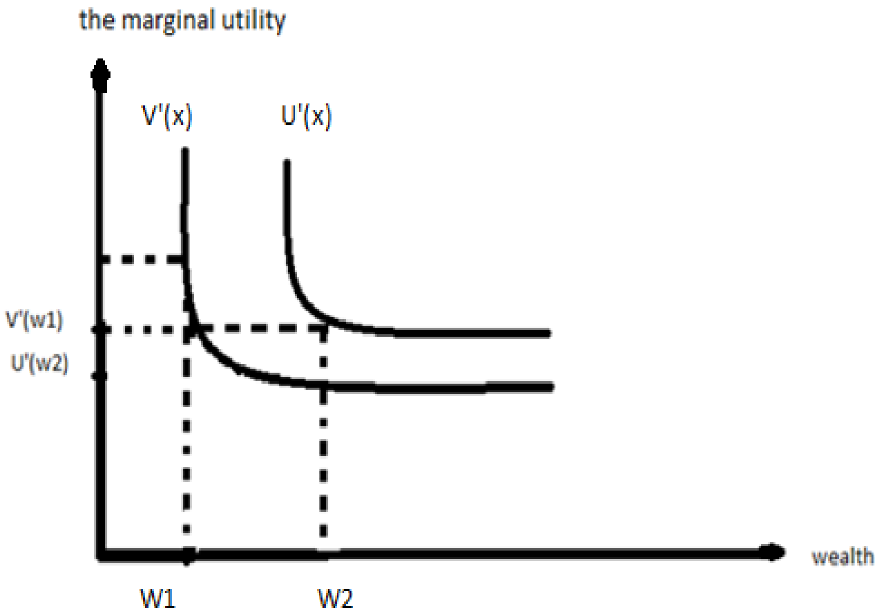
GRAPHIC: 6



It is therefore easy to show that:

$W_1 < W_2$

$(w_0 + (1-p)\alpha(u-v) - (u-v)) < (w_0 - \alpha(u-v)p)$

$(w_0 - p\alpha(u-v) + \alpha(u-v) - (u-v)) < (w_0 - \alpha(u-v)p)$

$\alpha(u-v) - (u-v) < 0$

$(\alpha - 1)(u-v) < 0$

Or $(u-v) > 0$

$(\alpha - 1)$ must be negative .

So $\alpha < 1$

$\alpha^* < 1$: The less than full insurance is optimal.