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Analysis of the EUR/USD Forward Exchange Premium

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ABSTRACT

In this paper, we adopt two different specifications of the ARCH / GARCH modeling, given its descriptive and predictive advantages, to analyze the EUR/USD forward exchange premium. In a first step, we estimate a symmetric linear model by taking into account the effect of the mean and the conditional variance in a univariate framework. In a second step, we proceed to estimate the AR (1) - GJR - GARCH and the AR (1) - GJR - GARCH -M models that fit into a linear, bivariate and asymmetric framework. The estimation results indicate that the shocks hitting the conditional variance are quite persistent over time and this can reveal the presence of regime switching in the process explaining the variance. In addition, they show the existence of an asymmetry in the dynamics of the conditional variance characterizing the three-month and the six-month forward premiums.

Keywords: *Asymmetry, Forward premium puzzle, GJR-GARCH, Regime switching, Subprime crisis.*

1. INTRODUCTION

One of the most puzzling characteristics of the attitude of exchange rates , since the advent of floating exchange rates in the early seventies , is shown by the tendency of countries with high interest rates to see their currencies appreciate rather than depreciate as suggested by the uncovered interest rate parity . This puzzle of the uncovered interest rate parity known as “the forward premium puzzle”, is prone to an abundant theoretical and empirical review of literature and it was a crucial phenomenon in the field of International Finance. Many studies in this area include the works of Bilson (1981), Cumby (1988), Fama (1984), Gregory and McCurdy (1984) and Hodrick and Srivastava (1984). The most widely accepted interpretation of the exchange returns forecasting is materialized by the existence of a time varying risk premium on the foreign exchange markets.

The research that considers that, in the context of balanced portfolios, the forward premium has not been successful by professionals is limited. Indeed, the net position of the majority of U.S. assets does not change sign with a sufficient frequency to explain the attitude of time varying risk premiums. Although the model of Carlson and Osler (2003) is positioned in the short term, it shares many properties with models with balanced portfolios, including the importance of net positions of international risk premiums assets. Both authors suggest that short-term assets are most appropriate for the forward premium puzzle, since the puzzle applies only to short-term forward premiums (Chinn and Meredith, 2002).

The rejection of the uncovered interest rate parity is an empirical regularity on the international financial markets. This relationship implies that the regressions of on the forward premium devaluations should lead to a slope coefficient equal to unity and an intercept equal to zero. Empirically, the slope coefficient estimates from such regressions are typically negative. The evidence implies that follows predictable investment income on the foreign exchange markets. Hodrick (1987), Baillie and McMahon (1989), Froot and Thaler (1990) and Engel (1996) have enriched the literature by their empirical evidences that have been explained by three main forward explanations. The first one is represented by the existence of rational and time varying risk on the foreign exchange markets. Thus it was difficult to find theoretical models of the risk premium that can replicate the regression results reported in the literature. Then it is to show the irrational behavior of the participants on the foreign exchange markets. The last explanation is based on the small sample properties of the used estimators (Baillie and Bollerslev (2000) and Bekaert and Hodrick (2000)).

Moreover, there is a large literature on the existence of time varying risk premiums on the foreign exchange market and its influence in explaining the difference between the forward exchange rate and the future spot exchange rate. In the same perspective, Bhar and Chiarella (2003) aim to model the risk premium as a mean reversion diffusion process. The approach was then allowed to use daily observations of forward exchange rates for different maturities. That is, in return, it is to characterize the risk premium over time for these maturities, and subsequently obtain a term structure of the risk premium for three different maturities of forward exchange rates. On the other hand, Frankel and Poonawala (2004) argue that the foreign exchange forward market is less biased for emerging currencies than for the case of base currencies. Indeed, several studies have replicated the result which states that the forward exchange rate is an unbiased predictor of the future spot exchange rate.

Following these developments, we propose, in this paper, to study the dynamics of the EUR / USD forward premium and its main features via a symmetric linear and univariate, and an asymmetric and bivariate ARCH / GARCH modeling. This choice is based on the works that argue that ARCH / GARCH models provide a better

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forecasting of low horizon variability characterizing the foreign exchange risk premium. First, we estimate a GARCH -in Mean model in which the conditional variance is supposed to explain the forward exchange premium. However, the quadratic specification in the conditional variance equation that characterizes the GARCH -M model conceals the asymmetric shocks. Given this, we will look at a variety of other nonlinear extensions that have been proposed, including the GJR-GARCH model of Glosten and al. (1993).

The remaining of the paper is organized as follows. Section 2 presents the modeling of the foreign exchange forward premium. Section 3 reports the Univariate analysis and the empirical results concerning the GARCH-in-Mean Model. Section 4 is devoted to the Bivariate analysis and reports the estimation results of the GJR-GARCH and GJR-GARCH-M models. Section 5 concludes.

2. MODELING THE FORWARD PREMIUM ON THE FOREIGN EXCHANGE MARKETS

To analyze the forward exchange premium, we specify the difference between the spot exchange rate and the forward exchange rate $(f_t^{t+1} - s_t)$ as the forward premium, we denote by :

- s_t : The natural logarithm of the spot exchange rate at time t
- f_t^{t+1}: The natural logarithm of the forward exchange rate at time t
- E_t (.): The expectations operator conditional on the information available at that date
- ε_t: A random term with zero mean.

2.1 Data

Our study focuses on the parity of the Euro against the U.S. Dollar. We examine daily observations, end of period, which are the spot and the three-month, six-month and one-year forward exchange rates. We have 2408 observations covering the period from 04/01/1999 to 26/03/2008. All time series are obtained from the DataStream database and are expressed in logarithmic form to avoid the Siegel's paradox (Baillie and McMahon, 1989).

2.2 The Graphical Analysis

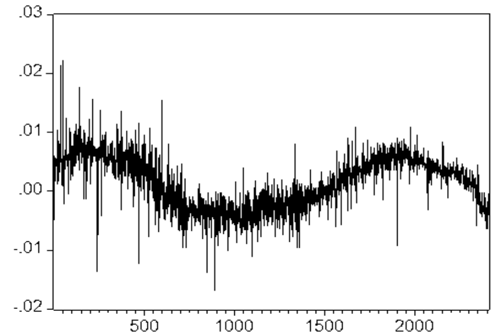

Fig 1: Graph of the EUR/USD 3-month forward premium

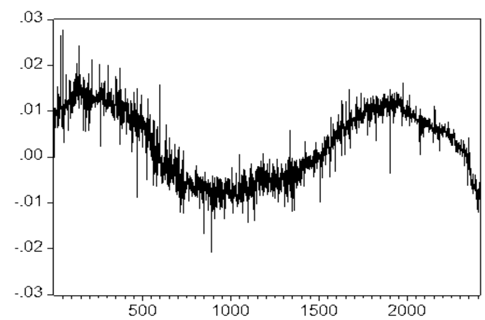

Fig 2: Graph of the EUR/USD 6-month forward premium

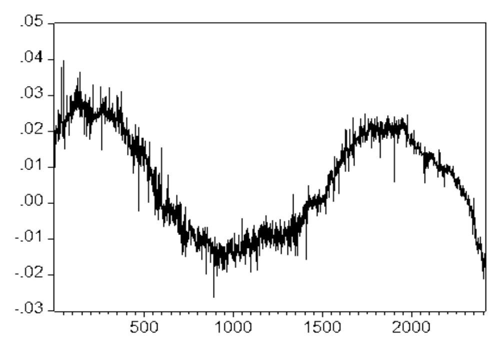

Fig 3: Graph of the EUR/USD 12-month forward premium

By examining the graphs (1), (2) and (3) of the Euro / U.S. Dollar three-month, six-month and one-year forward premiums over the period of study ranging from 04/01/1999 to 26/03/2008, we see remarkable fluctuations in terms of changes over time for each series. This leaves to suppose a priori either the stationary of the series or the hypothesis of persistent long memory to be confirmed with related tests.

These graphic illustrations are supposed to track better, at least in the time scale, the evolution phases of the EUR/USD forward premium throughout the study period adopted.

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Indeed, the introduction of the Euro as a single European currency has not only affected European countries, but it turns out that this currency plays an important role in the international monetary system. Since its creation on January 4, 1999, it is the "big bang" of financial markets because these markets in the euro area tilt entirely in Euro. Subsequently, the evolution “yoyo” of the Euro / U.S. Dollar exchange rate highlights the sharp fall of the Euro against the U.S. dollar between early 1999 and mid 2001. Following this, there was a trend appreciation of the Euro until the beginning of 2005. Further depreciation against the dollar occurs between early 2005 and early 2006. In addition, a long period of strong appreciation of the euro relative to the dollar starts early 2006 and is completed in early 2008; but the depreciation really starts in mid 2008.

Furthermore, the non-stationary data after December 2007 is due to the instability of exchange rate movements generated by the subprime crisis. This financial crisis is a crisis of subprime mortgages began in August 2007 and it is only about eight months that the EUR / USD forward exchange premium for which it is altered.

2.3 The Unit Root Tests

In order to test the stationary of the Euro / U.S. Dollar three-month, six-month and one-year forward premiums, we have used the unit root tests of Dickey and Fuller test (noted ADF) (1979, 1981), Elliot, Rothenberg and Stock (denoted ADF-GLS) (1996) and Kwiatkwski and al. test (denoted KPSS)(1992). The choice depended on testing ADF and ADF-GLS tests is based on the fact that they can test the validity of the null hypothesis of a unit root against the alternative hypothesis of no unit root. At this level, the disadvantage is that they show through due to the acceptance of the null hypothesis of unit root. As for the KPSS test procedure, it helps to overcome this problem by imposing the condition of stationary under the null hypothesis. In addition, the combined use of such tests can draw conclusions about the nature of the processes they are short memory and long memory.

We note that the ADF and ADF-GLS tests were conducted in the presence of levels of delay from 1 to 40 in the first differences of the series of the variables studied. Concerning the KPSS test, it was conducted in the window Newey-West (respectively that of Bartlett). In addition, the assumption about the presence or absence of a constant and a trend was also taken into consideration.

The results of the stationary tests are reported in Table (1.1).

Table 1.1: The unit root tests

	ADF Test H_0 : unit root		**ADF-GLS Test H_0 : unit root**		**KPSS Test H_0 : stationary**	
	In level	In 1st difference	In level	In 1st difference	In level	In 1st difference
Forward premium (3 months) EUR/USD						
Test statistic	-2.4461*** (10) [1]	-61.5077 (1) [1]	-2.3980*** (6) [1]	-19.3664 (1) [1]	1.1146*** [2]	0.1161 [2]
Critical value(1%)	-2.565927	-2.565927	-2.565926	-2.565926	0.216	0.216
Forward premium (6 months) EUR/USD						
Test statistic	-2.2368*** (5) [1]	-60.4702 (1) [1]	-2.0598*** (3) [1]	-20.0416 (1) [1]	1.0813*** [2]	0.1419 [2]
Critical value(1%)	-2.565925	-2.565924	-2.565924	-2.565924	0.216	0.216
Forward premium (12 months) EUR/USD						
Test statistic	-2.0528*** (2) [1]	-60.4044 (1) [1]	-1.9832*** (1) [1]	-21.0929 (1) [1]	1.021*** [2]	0.1498 [2]
Critical value(1%)	-2.565924	-2.565924	-2.565923	-2.565924	0.216	0.216
Spot exchange return						
Test statistic	-34.3060 (1) [1]	-58.8128 (1) [1]	-18.1373 (1) [1]	-53.5199 (1) [1]	0.1365 [2]	0.0393 [2]
Critical value(1%)	-2.565924	-2.565924	-2.565924	-2.565924	0.216	0.216

Note: Values in parentheses denote the number of lags used.
*, **, *** indicate that corresponding statistics are significant respectively at 10%, 5% and 1% levels.

Values in brackets indicate the type of model used for knowing the ADF test: The model (1): without constant. The model (2): with constant. The model (3): Constant and trend.

We note, in light of the results of unit root tests, that the EUR/USD forward premium series at 3 months, 6 months and 12 months horizons are not stationary at the 1% level significance; then we reject the hypothesis H_1 of stationarity of series. Moreover, referring to the calculated values of ADF, ADF-GLS and KPSS tests, we reject unambiguously the null hypothesis of a unit root in differentiated forward premium series whatever the model considered. The stationary nature of differentiated once series allows us to conclude an integration order equal to one. However, the spot exchange return series show a stationary which is maintained for different levels of delays of up to 20, in particular for the ADF test.

$$(f_{t,3} - s_t) \to I(1), (f_{t,6} - s_t) \to I(1), (f_{t,12} - s_t) \to I(1)$$
$$d(f_{t,3} - s_t) \to I(0), d(f_{t,6} - s_t) \to I(0), d(f_{t,12} - s_t) \to I(0)$$

The series considered are non-stationary and then they should be stationnarised (remove the deterministic component) by the method of Ordinary Least Squares (OLS).

We will be based in our empirical investigation on stationary series.

2.4 Descriptive Statistics

The Descriptive statistics relating to daily EUR/USD 3, 6 and 12-month forward premiums are shown in table (1.2).

Table 1.2: Descriptive statistics of forward premium series

	Forward premium (3 months)	**Forward premium (6 months)**	**Forward premium (12 months)**
Nb.observations	2407	2407	2407
Mean	-4.13^{e-06}	-8.26^{e-06}	-1.59^{e-05}
Median	0.0000	1.51^{e-06}	0.0000
Std.Dev	0.003086	0.003074	0.003067
Skewness (Sk)	0.042575	0.029015	0.052139
Kurtosis (Ku)	8.622270	7.569343	7.637303
Jarque-Bera (JB)	3170.938	2094.317	2157.821
Prob	0.0000	0.0000	0.0000
Q(12)	560.44	543.97	520.37
Q(24)	564.46	550.87	526.10

Statistics provided by Eviews 5.0

Inspection of Table (1.2) shows that the distributions of EUR/USD forward premiums (whatever the 3, 6 and 12-month horizon) are asymmetric showing skewness coefficients which are positive, then inducing thicker right series. We also note that there are indeed extreme values for all premiums eventually studied, since the skewness and their respective averages have opposite signs. This shows in particular that the Euro met phases of sudden depreciation and appreciation respectively.

About the kurtosis coefficient of 3, 6 and 12-month forward premium series, it is higher than the reference value of the normal distribution equal to 3. We then deduce that the distribution of the forward premium of the euro against the dollar is leptokurtic, then having a thicker tail than that of the normal distribution.

Given the analysis above - mentioned, it is not surprising that the null hypothesis of normality is strongly rejected by the asymptotic Jarque-Bera (1980) test for the EUR/USD forward premiums. Indeed, the JB statistic is much higher than the critical value given by the Chideux table with two degrees of freedom equal to 5.99 at the 5% level significance. Eventually, these normality tests have helped us to prove some heteroscedasticity materialized by leptokurtic distributions, and thereby confirming that it is of volatile variables.

Regarding the Q statistic, it is distributed asymptotically as a Chideux (at 12 and 24 degrees of freedom). We note clearly, from this table, all Q Ljung-Box statistics are above $\chi^2(20)$ read in the table at 5% level significance and with a value of 31.41. Also, they clearly indicate, by their critical zero probabilities, series of forward premiums unrepresentative of white noise. They also indicate that these series demonstrate significantly from a phenomenon widely known as the volatility clustering, which is ultimately linked to the notion of heteroscedasticity.

At this stage, it is important to note that the existence of non-linearity can be explained either by the presence of ARCH effect, or by the existence of a long memory.

3. UNIVARIATE ANALYSIS

Hereafter, we propose to estimate the GARCH-M model (p, q) defined as follows:

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t \tag{1.1}$$

$$h_t = a_0 + a_1 h_{t-1} + b_1 \varepsilon_{t-1}^2 + \eta_t \tag{1.2}$$

$$(\varepsilon_t / t) \sim N(0, h_t) \tag{1.3}$$

Equation (1.1) is the equation of the conditional mean.
Equation (1.2) is the equation of the conditional variance.
Equation (1.3) is the assumption of conditional normality of errors.

- Ht: is the conditional variance of forward premium series which is assumed to follow a GARCH (1,1) process.
- Ht-1: represents the forecasting of the variance at the last period and the coefficient a_1 associated therewith represents the GARCH parameter.
- ε^2_{t-1}: represents the squared delayed residuals informing us about the volatility or the instantaneous variability, and the coefficient b_1 associated therewith is referred to the ARCH parameter.

We note that equation (1.1) is the pivotal equation of GARCH-M model in which the forward exchange premium is a function of its conditional variance. In this specification in mean, the conditional variance is introduced into the mean equation and the choice of such a model depends on its ability to capture stylized facts of forward exchange premiums (at low or high frequency).

3.1 The GARCH-in-Mean Model

In order to apply the GARCH -in -Mean specification for the EUR/USD 3, 6 and 12-month forward premium series, we should firstly check certain conditions that will allow us to confirm the use of such a heteroscedastic model in which there is inevitably a volatility effect. Indeed, the ARCH-type models can model chronics that have an instantaneous volatility depending on the past, and it will then be possible to develop a dynamic forecasting of the exchange risk premium in terms of mean and variance.

After the descriptive statistics of the forward exchange premiums, we note that it is possible to estimate these series via the GARCH -M model since they are neither normal distributions nor white noise processes, as required by the heteroscedastic ARCH specification.

However, it should be checked prior to the adequacy of the GARCH -M model (p, q) for different orders. For this, we use the statistical criteria of Akaike (AIC) and Schwarz (SC) to determine the optimal pair (p,q) that minimizes both of these two functions :

Table 1.3: GARCH (p,q)

	EUR/USD	
(p,q)	(1,1)	(1,2)
AIC	-8.919577	-8.919103
SC	-8.909962	-8.907084

Extracted from the software Eviews 5.0

With regard to the table (1.3), we find that the adequate order of the GARCH-M model that minimizes both the criteria of Akaike and Schwarz is relative to the pair (1, 1).

3.2 Estimation Results

The estimation of GARCH-M (1,1) model respectively for the EUR/USD three-month, six-month and one-year forward premiums is equivalent to estimating both the mean equation and the conditional variance equation for these series.

The estimation results of the GARCH-M (1,1) model are shown in Table (1.4).

Table 1.4: Estimation of GARCH-M (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$
$$h_t = a_0 + a_1 h_{t-1} + b_1 \varepsilon_{t-1}^2 + \eta_t$$
$$(\varepsilon_t / t) \sim N(0, h_t)$$

	Forward premium 3 months	Forward premium 6 months	Forward premium 12 months
β	-3.416949 (5.068835)	-2.951398 (5.340551)	-0.520904 (5.539467)
a_0(constant)	4.16E^{-06}* (2.01E^{-07})	3.32E^{-06}* (1.97E^{-07})	2.88 E^{-06}* (2.15E^{-07})
b_1(ARCH)	0.410064* (0.035898)	0.3677892* (0.034085)	0.357636* (0.034405)
a_1(GARCH)	0.164509* (0.031239)	0.291047* (0.03193)	0.347746* (0.039110)
Q (20) Q^2 (20)	$365.82_{[0.000]}$ $46.674_{[0.001]}$	$379.41_{[0.000]}$ $38.946_{[0.007]}$	$362.61_{[0.000]}$ $40.268_{[0.005]}$
Jarque-Bera	4596.509	2434.706	2381.758

Estimates made on EVIEWS software (version 5.0)

Note: Values in parentheses are standard deviations and values in brackets are the p value of the null hypothesis of no autocorrelation of errors (Q) and their squares (Q^2). The superscript * indicates that the coefficient is statistically significant.

In the light of the table (1.4), the GARCH –M (1,1) estimation results show that the coefficient β of the mean equation is negative and statistically significant for each forward premium studied. This clearly shows that the EUR/USD three-month, six-month and one-year forward premiums are not explained in large part by their conditional variances, i.e by their volatilities. However, the constant of the variance equation and the ARCH and GARCH terms are meaningful for all series studied. We also note that the sum of the ARCH and GARCH parameters amounts to (0.5745, 0.6588 and 0.7053), respectively for the horizons of 3 months, 6 months and 12 months. Such values , not close to unity, are certainly indicative that the shocks induced by the volatility are not very persistent, so that forecasts of the conditional variance does not converge very slowly to the regular state.

With regard to the "Ljung-Box" test as applied to simple standardized residual series, Q (20) statistics are significant because the probabilities associated with each of the autocorrelations are below 0.05, which confirms the rejection of the assumption of normality for residual series, in addition to the fact that they are unrepresentative series of white noise. Thus, there is a residual ARCH effect not captured by the model.

In addition, the examination of the Ljung-Box statistic Q applied to the squared standardized residuals series shows that it is significant regardless of the study horizon. Therefore, we conclude that the squared residual series exhibit ARCH effects which are not taken into account in the model.

However, it remains to verify the normality of the distribution of standardized residuals. To do this, we examine the Jarque-Bera statistic and we see it clearly exceeds the critical value of χ^2 (2) at the level of significance of 5%. Therefore, there is rejection of the hypothesis of a normal residual distribution.

Finally, we conclude that the GARCH-M specification is not a good specification for the EUR/USD three-month, six-month and one-year forward premiums. Furthermore, we note that the estimation results of this model could be even better and insightful working with a database of high frequency such as intraday data.

It is hardly worth remembering that contemporary models of exchange rates time series make the use of GARCH widespread. Indeed, these models seem to not only capture the "volatility clustering", but also accommodate some leptokurtic characteristics (fat tails) usually found in the time series of the exchange rates and therefore in the forward exchange premiums. In addition, using the framework of ARCH models can account for heteroscedasticity that characterizes exchange rates and explain the forward premium by the volatility of profitability on the foreign exchange market.

4. BIVARIATE ANALYSIS

The GARCH-M model is among the linear models based on a quadratic specification of disturbances on the conditional variance. They assume that the magnitude and not the sign of the shock that determines the volatility. Therefore, positive and negative shocks of the same size have the same impact on the conditional variance. In other words, they are symmetrical process. However, the asymmetric efficiency of shocks on the volatility, i.e the conditional variance reacts differently to shocks of the same magnitude as the sign of the latter is very realistic for financial and monetary series. Symmetric ARCH models have the disadvantage of not taking into account the stylized fact possible in the series studied.

The inadequacy of GARCH-M process for the representation of the EUR/USD forward premium dynamics for horizons of 3 months, 6 months and 12 months leads us to consider a specification that is characterized by detection of asymmetry shown by the majority of financial series.

In what follows, we model the EUR / USD forward premium using the AR (1)-GJR-GARCH (1,1) model. Therefore, we move from GARCH-M model which is within the framework of linear ARCH / GARCH models to the application of asymmetric ARCH / GARCH models[1] such as GJR-GARCH and GJR-GARCH-M models.

4.1 The AR (1)-GJR-GARCH(1,1) Model

Under the univariate ARCH models, the model of Glosten et al. (1993), known by the abbreviation GJR, describes the asymmetry by distinguishing between two types of shocks that may affect the price of an asset that is a positive return not anticipated and an unanticipated negative return.

Another approach to capture the effect of asymmetric disturbances on the conditional variance is introduced by Glosten, Jagannathan and Runkle (1993). The GJR - GARCH formulation is in fact a GARCH model with the addition of a dummy variable which is multiplied by the square of the error term of time spent in the conditional variance equation. It is a threshold model where the indicator function, that is the dummy variable, is equal to one if the residual of the previous period is negative and it is zero otherwise. In this way, the conditional variance follows two different processes depending on the sign of the error terms.

Consider $\varepsilon_t = Z_t\sqrt{h_t}$, the equation for the conditional variance of a GJR-GARCH process is:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, h_{t-j} + \gamma\varepsilon_{t-1}^2 I_{t-1} \quad (1.4)$$

Where I_{t-1} = 1 if $\varepsilon_{t-1} < 0$, 0 if not

With the conditions $\alpha_0 > 0$, $\alpha_i, \beta_j \geq 0$ and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j + 0.5\gamma < 1$.

[1] Among these models, we include the EGARCH model, GJR-GARCH, and APARCH VSGARCH, Tarch and TGARCH, QGARCH, LSTGARCH and ANSTGARCH.

The GJR-GARCH (p,q) model captures the asymmetric effect of the disturbances on the conditional variance.

For further empirical investigation in this contribution, we propose to move to the modeling GJR-GARCH-in Mean, in order to integrate the conditional variance in the variance equation of the GJR-GARCH model.

We specify that the estimations of the GJR-GARCH model for the EUR/USD 3-month, 6-month and 12-month forward premiums are made based on the algorithm BHHH (1974).

4.2 Measurement of the Persistence of Volatility Shocks

We are trying to identify the model that best characterizes the EUR / USD forward premium. To do this, we will take a comparison between the GJR-GARCH (1,1), the AR (1)-GJR-GARCH (1,1) and the AR (1)-GJR-GARCH-M (1,1) processes.

We consider the following models to analyze the volatility of the forward exchange premium.
The estimation results of the AR (1)-GJR-GARCH (1,1) model and the AR (1)-GJR-GARCH-M (1,1) model are shown in Tables (1.5) and (1.6).

Table 1.5: Estimation of the AR (1)-GJR-GARCH (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$

	Forward premium (3 months)	Forward premium (6 months)	Forward premium (12 months)
Constant (M)	$-5.2099E^{-05}$ (-1.02719)	$-7.1456E^{-05}$ (-1.48094)	$-7.0589E^{-05}$ (-1.44972)
AR(1)	-0.4691* (-29.02337)	-0.4686* (-26.03182)	-0.4485* (-27.80787)
Constant (V)	$1.0945E^{-07*}$ (9.09568)	$3.0688E^{-07*}$ (10.39794)	$3.7076E^{-07*}$ (11.43284)
Arch	0.0326* (5.03214)	0.0725* (6.063)	0.1045* (7.04804)
Garch	0.9351* (282.32773)	0.8568* (89.57063)	0.8318* (84.38984)
D	0.0385* (3.26025)	0.0710* (2.91623)	0.0422 (1.52166)
Q (20) Q^2 (20)	$525.331_{[0.064]}$ $83.772_{[0.42]}$	$543.629_{[0.052]}$ $67.984_{[0.67]}$	$523.447_{[0.048]}$ $87.556_{[0.57]}$

Estimate made by Rats 7.0 software

Note: The digital resolution was achieved via the algorithm BHHH (Berndt, Hall, Hall and Hausman, 1974).
D is the skewness.

The values in parentheses are the t-Student statistics and the values in braces are p.value of the null hypothesis of no autocorrelation of errors (Q) and their squares (Q^2). The exponent * indicates that the coefficient is statistically significant.

Based on the estimation results of the forward premiums, we can affirm that the estimated model has good statistical properties. Indeed, in light of the table (1.5), the estimation results of the AR (1) - GJR - GARCH (1,1) model with the presence of a normal distribution indicate that the estimated coefficients of Arch and Garch parameters have the same sign and are statistically significant (with a term Garch demonstrating a strong significance) . In addition, the sum of these two parameters is very close to unity (0.9677, 0.9293, and 0.9363 respectively for the horizons of 3 months, 6 months and 12 months), so it is indicative that the shocks that are imposed on the conditional variance are quite persistent over time. The high persistence of shocks of the conditional variance can reveal the presence of regime shifts in the process explaining the variance. Moreover, the autoregressive effect of order 1 is strongly present and showed significance given that the estimated parameter AR (1) is negative and statistically significant for all maturities.

The analysis of the conditional variances equations confirms the existence of an asymmetry in the dynamics of the conditional variance. Indeed, the coefficient D related to the non-linear (or asymmetric) component of the conditional variance and referring to the leverage is positive and statistically significant for the EUR / USD forward premiums for the horizons of 3 months and 6 months. Contrariwise, the GJR - GARCH effect is completely absent for the 12-month forward premium. This confirms the adequacy of the AR (1) - GJR - GARCH (1, 1) process in our case. Thus, the forward premium is characterized by an asymmetry highlighted by the coefficient D. Therefore, an increase of the conditional variance is associated with an increase of the conditional premium.

The test for the presence of ARCH effects in the standardized residuals, the probabilities of the Q statistics are greater than 0.05, so they reveal that the studied series are representative of white noise series.

Subsequently, we estimate the AR (1)-GJR-GARCH-M (1,1) model, whose results are shown in Table (1.6).

Table 1.6: Estimation of the AR (1)-GJR-GARCH-M (1,1) model

$$f_t^{t+1} - s_t = \beta h_t + \varepsilon_t$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, h_{t-j} + \gamma \varepsilon_{t-1}^2 I_{t-1}$$
$$(\varepsilon_t / t) \sim N(0, h_t)$$

	Forward premium (3 months)	Forward premium (6 months)	Forward premium (12 months)
Constant (M)	-0.000194* (-2.07067)	-0.000218* (-2.70174)	-0.000195* (-2.43754)
AR(1)	-0.469452* (-28.68535)	-0.469324* (-26.16280)	-0.448546* (-27.76146)
β	25.038856 (1.74081)	26.396437* (2.12919)	22.298749 (1.78661)
Constant (V)	0.0000* (9.21331)	0.0000* (9.85603)	0.0000* (10.61278)
Arch	0.031755* (4.88914)	0.065816* (5.77291)	0.097327* (6.44183)
Garch	0.926860* (235.83479)	0.849361* (79.20838)	0.827099* (73.31486)
D	0.047725* (3.92942)	0.088123* (3.56805)	0.059514* (2.08372)

Estimate made by Rats 7.0 software

Note: The digital resolution was achieved by the algorithm BHHH (Berndt, Hall, Hall and Hausman, 1974).
D is the skewness.

Values in parentheses are the t- Student statistics.

The superscript * indicates that the coefficient is statistically significant.

On the other hand, the GJR - GARCH -M model is a good specification for EUR / USD forward premium series only for the 6-month horizon because the β coefficient of the conditional mean is positive and statistically significant. Similarly, there is a high persistence of shocks of the conditional variance as the sum of the Arch and Garch parameters is close to unity (0.9151). In addition, the asymmetry is remarkably present in this system of the fact that the D coefficient is positive and statistically significant. The autoregressive effect is also strongly required. All these features of the EUR/USD six-month forward premium are valid for the horizons of 3 months and 12 months. However, the GJR - GARCH in Mean effect is totally absent for these two horizons because the β coefficients are not significantly different from zero.

Overall, given the findings mentioned above, we deduce that the AR (1) - GJR - GARCH (1,1) with a normal distribution turns out to be a good specification of the EUR/USD forward premium. This asymmetric and bivariate analysis is certainly more intuitive and advantageous than the univariate analysis based on symmetric GARCH -M linear modeling neglecting the asymmetry that can take the forward premium on the foreign exchange market.

5. CONCLUSION

In this paper, we aimed to describe the dynamics of the EUR/USD forward premium on the foreign exchange markets and to study the behavior of the forward exchange premium, which constituted, through its mixed and conflicting results, a famous puzzle. Indeed, the forward premium puzzle has for a long time baffled both the financials and the economists.

Given this, we found it would be interesting to test the relevance of the heteroscedastic GARCH -in –Mean model in the estimation of the forward premium on the international foreign exchange markets. Our empirical analysis has also been based on the GJR - GARCH model which is a relatively intuitive modeling based on the asymmetry assumption of the volatility.

The choice of this asymmetric model is more appropriate and parsimonious for the study of financial time series since the estimation of this class of models shows a high degree of persistence of the conditional variance. The presence of structural changes in the conditional variance following a major event could bias the high degree of volatility. According to Diebold (1986), the high persistence in ARCH models is due to the presence of abrupt changes and the standard GARCH models then overestimate the true value of the process variance. This explanation of Diebold has been tested and confirmed empirically by Lamoureux and Lastrapes (1990) who found that the introduction of a deterministic regime switching in the conditional variance to reduce a remarkable level of persistence of the conditional variance compared to that given by the ordinary GARCH models.

The comparative analysis in the context of this class of models is in favor of the AR (1)-GJR-GARCH model to retrace at best the EUR/USD forward premiums. This seems to be legitimate given the inability of ARCH / GARCH standard models to reflect some phenomena such as cyclic oscillatory behavior, the abrupt shocks and the asymmetry of the series volatility.

REFERENCES

[1] Backus, D.K., Gregory, A.W. and Telmer, C.I. (1993), “Accounting for forward rates in the forward premium anomaly”, Journal of Finance, 56, pp. 279-304.

[2] Baillie, R.T. and Bollerslev, T., (2000), “The forward premium anomaly is not as bad as you think”, Journal of International Money and Finance, 19, pp. 471-488.

[3] Baillie R.T and Kiliç R., (2006), « Do asymmetric and non linear adjustments explain the forward premium anomaly?”, Journal of International Money and Finance, 25, pp. 22-47.

[4] Bansal R., (1997), “An exploration of the forward premium puzzle in currency markets”, The Review of Financial Studies, 10 (2), pp. 369-403.

[5] Bansal, R., and Dahlquist, M. (2000). “The forward premium puzzle: Different tales from developed and emerging economies”, Journal of International Economics, 51.

[6] Bekaert, G., and Hodrick, R. (1993). “On biases in the measurement of foreign exchange risk premiums”. Journal of International Money and Finance, 12.

[7] Bernoth K., Hagen J. and Vries C.G., (2007), « The forward premium puzzle only emerges gradually », Tinbergen Institute Discussion Paper n° 033/2.

[8] Boudoukh J., Richardson M. and Whitelaw R., (2005), “The information in long-maturity forward rates: implications for exchange rates and the forward premium anomaly”, NBER Working Paper n° 11840.

[9] Chakraborty A. and Evans G.W., (2008), “Can perpetual learning explain the forward premium puzzle?”, Journal of Monetary Economics, 55, pp.477-490.

[10] Ding L., (2005), “Term structure of forward premium puzzles”, department of economics, University of North Carolina.

[11] Duarte M. and Stockman A.C., (2005), “Rational speculation and exchange rates”, Journal of Monetary Economics, 52, pp.3-29.

[12] Engel, C. (1996). “The forward discount anomaly and the risk premium: a survey of recent evidence”, Journal of Empirical Finance, 3.

[13] Engel, C., and Kim, C. J. (1999). “The long run US/UK real exchange rate”, Journal of Money, Credit and Banking, 31.

[14] Engle, C., and Hakkio, C. S. (1996). “The distribution of Exchange rates in the EMS”, International Journal of Finance and Economics, 1.

[15] Fama, E. (1984). “Forward and spot exchange rates”, Journal of Monetary Economics, 14.

[16] Farhi E. and Gabaix X., (2007), « Rare disasters and exchange rates: a theory of the forward premium puzzle », Harvard University, NYU Stern and NBER.

[17] Frenkel, J. A. (1980). “Exchange Rates, Prices and Money: Lessons from the 1920s”, American Economic Review, 70.

[18] Frenkel, J. A., and Froot, K. A. (1989). “Forward Discount Bias: Is It an Exchange Risk Premium?”, Quarterly Journal of Economics, 104.

[19] Froot, K. and R.H. Thaler, (1990). “Anomalies: Foreign exchange”, Journal of Economic Perspectives, 4.

[20] Groen J.J and Balakrishnan R., (2006), «Asset price based estimates of sterling exchange rate risk premia, Journal of International Money and Finance, 25.

[21] Jongen R., Verschoor W. and Wolff C., (2008), “Foreign exchange rate expectations: Survey and synthesis”, Journal of Economic Surveys, 22(1), pp.140-165.

[22] Kellard N. and Sarantis N., (2008), “Can exchange rate volatility explain persistence in the forward premium?”, Journal of Empirical Finance, 15, pp.714-728.

[23] Mark N.C. and Moh Y.K., (2007), « Official interventions and the forward premium anomaly », Journal of Empirical Finance, 14, pp.499-522.

[24] Yang K. and Shintani M., (2006), « Does the prediction horizon matter for the forward premium anomaly? Evidence from panel data”, Economics Letters, 93, pp.255-260.