

The Implications of Catastrophe Theory for Stock Market Forecasting

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ABSTRACT

This paper aims to study a controversial issue such as the implications of Catastrophe Theory for stock markets forecasting. Stock markets are extremely unpredictable and complicated, so it's difficult to believe that their chaotic behavior can be classified into a specific pattern. In general, the concept of stock market is characterized by dramatically movements, nonlinearity, uncertainty, anomalies and cycles of evolution. Catastrophe Theory is the study of sudden changes in a system, which in this case is represented by the stock market that results from smooth and insignificant changes in those factors that determine the equilibrium state of this system. Thus, Catastrophe Theory explains in particular sudden changes from one equilibrium state to another. In other words, Catastrophe Theory has established a new level of understanding regarding the concept of stock market.

Keywords: *Catastrophe Theory, stock market, disequilibrium, discontinuities*

1. INTRODUCTION

Stock market is characterized by complex nonlinear dynamics which does not converge to a known purpose, a result that may be anticipated or influenced, or at least to a limit cycle. In this context, even the concept of financial investment rationality reaches a completely different meaning. Thus, limited rationality replaces rational expectations. At a certain level, on the stock markets is distinguished exclusively the human dimension of the financial events and, of course, the causes that led to the triggering of these particular phenomenon. Catastrophe Theory is applied to explain different categories of discontinuities. In recent past, this unconventional theory was applied in the financial field for modeling discontinuities based on complex and quite sophisticated mathematical models. Catastrophe Theory provide a certain type of approach that derives directly from pure mathematics, but also combines in a smooth and paradoxical manner, elements from other sciences such as biology, embryology, philosophy or geophysics. Catastrophe Theory is perceived as a theory of forms. Moreover, its quintessence derived from the development of mathematical models based on the idea of the existence and stability of forms, respectively, on the morphogenesis. Also, Catastrophe Theory provides a distinct methodology to study discontinuous changes and qualitative leaps. It is interesting to emphasize that the qualitative and quantitative dimensions are not antagonistic, but complementary. This unconventional theory is used mainly to describe a no stationary disequilibrium process in the stock market system. The analysis of stock market based on Catastrophe Theory provides a completely different perspective compared to traditional finance paradigm. Consequently, human behavior and thus financial investment decisions must be seen through the interaction between the dynamics of quantitative and qualitative structures.

2. LITERATURE REVIEW - CATASTROPHE THEORY – A REVOLUTIONARY AND CONTROVERSIAL CONCEPT

Catastrophe Theory was developed by the French mathematician René Thom, in the late 1960s, being focused on a rational approach to the phenomenon of discontinuous change in behaviors, namely outputs, determined by the continuous change in parameters, namely inputs, in a given system. Its fundamental principles were exhibited for the first time in Thom's article "A Dynamical Theory of Morphogenesis" published in 1968, and later in his essential and brilliant book "Structural Stability and Morphogenesis" which has been available since 1972. However, Thom's research was influenced in a fundamental manner by the British mathematician Christopher Zeeman known for his work in geometric topology and singularity theory. This article is primarily an attempt to identify the implications of Catastrophe Theory for stock market forecasting, despite the fact that this theory is perceived as "a state of mind" [R. Thom, 1976] and "a beautiful, intriguing field of pure mathematics" [D. Castrigiano and S. Hayes, 1993]. The central idea is that although discontinuities may arise dynamically due to random exogenous shocks, the most fascinating and dramatic ones are those that arise endogenously from dynamical economic systems, and especially the stock market. Moreover, in dynamical systems these discontinuities can arise endogenously in the presence of nonlinearities. What is striking is that discontinuity is seen to contrast sharply with what have been standard approaches in financial theory quite generally. Accordingly, Catastrophe Theory is in sharp contradiction with Efficient Market Hypothesis. A market in which prices always "fully reflect" available information is called efficient. The most important issue regarding efficient market theory is that it is not possible to outperform the market over the long-term. Catastrophe Theory is a unique collage made up of different mathematical structures, such as : singularities of smooth mappings, bifurcation theory, qualitative dynamycs,

stratified spaces, singularities of differential forms and many others besides these. In addition, this theory provides certain typologies of explanations that are directly inspired by structuralism, morphogenesis and topology. Metaphorically, Catastrophe Theory represents a fascinating interdisciplinary connector which may provide an explanation for the occurrence of unexplained events such as stock market crashes or other severe financial events. According to some researchers, Catastrophe Theory represents an incomplete transition from explanations in terms of a few simple, stable forms to an understanding of nature in terms of the complex, the fluid and the multiple. Obviously, from a certain point of view, it may be possible to perceive Catastrophe Theory as a crucial transitional stage from structuralism to poststructuralism, perhaps even from modernism to postmodernism [D. Aubin, 1997]. The quintessence of René Thom's research is that provides the modeling of complex phenomena with a methodology founded on the analysis of some universal morphology, namely the catastrophes. In the field of stock markets, the term catastrophe acquires a special resonance. Thom suggested that : "From my viewpoint, Catastrophe Theory is fundamentally qualitative, and has as its fundamental aim the explanation of an empirical morphology. Its epistemological status is the one of an interpretative-hermeneutic theory. Hence it is not obvious that it will necessarily develop into new pragmatic developments." Stock markets are very complex and unpredictable. Stock markets' chaotic behavior and non-linearities, complication and uncertainty, unexpected booms and crashes are some of the most challenging problems. The stock market is a complex and dynamic system with noisy, no stationary and chaotic data series (Peters, E. E, 1994). Some researchers are suggesting that such complexity is an intrinsic characteristic of such system. The interesting thing about the chaotic dynamics of stock markets is its great ability to generate dramatic movements which appear to be random, with higher frequency than linear models. Obviously, this chaotic behavior of stock markets is a non-linear deterministic process, because linear models can only generate four types of behavior: oscillatory and stable, oscillatory and explosive, no oscillatory and stable and oscillatory and explosive (Hsieh, A. D., 1991). The non-linear models are much more complex and can generate a much more varied types of behavior. The purpose of this article is to discuss some of the methodological issues in stock markets modeling. The stock market has always been one of the most popular investments due to its high returns (Kuo, R.J., Lee, L.C., Lee, C.F., 1996). Because of that, stock market modeling is an area of interest to stock traders, quantitative finance specialists, investment professionals and applied researchers. One of the most controversial issue in the recent past is probably whether the stock market can be predict in a satisfactory manner or not. However, there is always some risk to investment in the stock market due to its unpredictable movements. Thus, an appropriate prediction model for stock market forecasting would be highly valued and useful, being an issue of a major interest [Birău, R., 2011]. The influence

of informational asymmetry, psychological, sociological and demographic structures can represent up to a certain level, a relevant answer to financial market anomalies. Investors are different some of the other in relation to numerous factors, such as : socio-economic background, financial context, level of education, religion, age, sex, traditions, ethnicity, marital status, and so on. They form expectations and beliefs that influence their investment decisions in a dramatic proportion. An optimum investment decision it cannot be achieved if the investor ignores all of these factors influence and has a unrealistic expectations of stock market.

3. A VISIONARY METHODOLOGY

Catastrophe theory is focused on degenerate critical points of the potential function, points where not just the first derivative, but one or several higher derivatives of the potential function are also zero. These particular points are known as the germs of the catastrophe geometries. The degeneracy of these critical points can be unfolded by expanding the potential function as a Taylor series in insignificant perturbations of the parameters. Thom identified two categories of points: those called "regular", corresponding to the areas of continuity of the morphogenetic process, and those called "catastrophic", where the phenomenological appearance of the substratum abruptly changes. The importance of these points derive from the fact that in catastrophic points, the morphogenetic process exhibits a discontinuity. Thom concluded that Catastrophe Theory is reduced in the opinion of the majority of people only to a certain portion of the conglomerate, namely "elementary catastrophe theory" involving the seven catastrophes on R^4 . The basic schema of an "elementary catastrophe" follows a specific methodology that will be presented below. Suppose we have a system (S) the states of which are parameterized by a point ξ in a smooth manifold M. Suppose that we may act on this system by varying a point u in a space U of control parameters. Suppose this determination is defined by a map (system of equations) :

$$(\Sigma) \quad F(\xi, u) = 0$$

$$\text{or} \quad F_i(\xi_j, u_k) = 0,$$

where $i = 1, 2, \dots, n$; $j = 1, 2, \dots, n$;

$$k = 1, 2, \dots, m.$$

Is obviously that the emphasis is on the fact that in all cases considered in control theory, one admits that this system of equations may be solved-through the implicit function theorem, with the assumption that the Jacobian $D(F_1, \dots, F_n) / D(\xi_1, \dots, \xi_n)$ is not zero. If $n = 1$ then (Σ) reduces to only one equation $F(x, u_i) = 0$. If $\partial F / \partial x \neq 0$ then we are in the standard situation (τ) . Elementary catastrophe theory is

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characterized, on the contrary, by situations where the transversality condition (τ) fails and such situations cannot be avoided in general [R., Thom, 1976]. Zeeman suggested in his article "Catastrophe Theory" that the seven elementary catastrophes, called the fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, and parabolic umbilic, describe all possible discontinuities in phenomena by no more than four factors. Therefore, each of the catastrophes is associated with a potential function in which the control parameters are exposed in the form of coefficients: a, b, c and d. Also, the behavior of the system is determined by the variables x and y. In the light of the Catastrophe Theory, stock market forecasting appears to be a game of dominoes with real parts and implications. The stake of this game is also real and often reached global dimensions. Zeeman made an attempt to provide an explanation of stock market dramatic and unpredictable behavior based on Catastrophe Theory. He emphasized that stock markets are known for terms such as "bull market" and "bear market" which suggest an obvious bimodality. Accordingly, a severe event like a crash or a financial collapse of the stock market can be classified as a catastrophe. Zeeman proposed a stock market model based on the fact that a "bull market" is a market with some excess demand and a high proportion of speculators, while a "bear market" is a falling market which discourages speculation, but after a certain period of time the resulting undervaluation encourages long-term investment. Thus, as a consequence, if the confidence level grows and involves excess demand, the index signs fragile and slowly increase, without catastrophe. Thereby, speculation is encouraged and investment discouraged because a new financial cycle begins.

4. CONCLUSIONS

One of the most controversial issue in the recent past is probably whether the capital market can be predict in a satisfactory manner or not. As the stock market environment is a very important area, it is a priority to be interested in developing more accurate forecasting models. Human understanding of stock market is limited, because our capability in analyzing all the data has not been complete and empirical economic methods have not been satisfactory. The obvious complexity of the stock markets has been investigated by various researchers and a large amount of research papers has been published in recent times. Resolving such complexity has been for many long years just an utopia. René Thom suggested that: "Perhaps the most interesting feature of Catastrophe Theory is its ability to describe analogies. In some sense, Catastrophe Theory could be called the theory of analogies, by associating with a local situation a singularity of the local dynamics; it provides an algebraic way of formalizing the intuitive notion of analogous situations." Catastrophe Theory represents a fascinating interdisciplinary connector which may provide an explanation for the occurrence of unexplained events such as stock market crashes or other severe financial events. This theory is primarily a vision, an unique and

controversial conglomerate of ideas that triggered a real revolution in science. This theory states that a catastrophe occurs when a continuous variation of causes involves a discontinuous variation of effects. The implications of Catastrophe Theory for stock market forecasting reach unfinished resonances, still unknown and unexplored properly. The practical limits of Catastrophe Theory regarding stock market are evident, but this is not a sign of the imperfection or the immaturity of the theory. Catastrophe Theory is not an experimental theory, but it is neither just a mathematical theory. It is a piece of a whole...perhaps one of the key pieces.

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Table 1: The seven elementary catastrophes

<i>Catastrophe</i>	<i>Control dimensions</i>	<i>Behavior dimensions</i>
Fold	1	1
Cusp	2	1
Swallowtail	3	1
Butterfly	4	1
Hyperbolic	3	2
Elliptic	3	2
Parabolic	4	2

<i>Catastrophe</i>	<i>Function</i>	<i>First derivative</i>
Fold	$\frac{1}{3}x^3 - ax$	$x^2 - a$
Cusp	$\frac{1}{4}x^4 - ax - \frac{1}{2}bx^2$	$x^3 - a - bx$
Swallowtail	$\frac{1}{5}x^5 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3$	$x^4 - a - bx - cx^2$
Butterfly	$\frac{1}{6}x^6 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3 - \frac{1}{4}dx^4$	$x^5 - a - bx - cx^2 - dx^3$
Hyperbolic	$x^3 + y^3 + ax + by + cxy$	$3x^2 + a + cy$ $3y^2 + b + 2cy$
Elliptic	$x^3 - xy^2 + ax + by + cx^2 + cy^2$	$3x^2 - y^2 + a + 2cx$ $- 2xy + b + 2cy$
Parabolic	$x^2y + y^4 + ax + by + cx^2 + dy^2$	$2xy + a + 2cx$ $x^2 + 4y^3 + b + 2dy$

Source: Zeeman, E.C. - Catastrophe Theory - Selected Papers 1972–1977, Reading, MA: Addison-Wesley, 1977